The 2nd Shot of The 13th MSJ-SI
"Differential Geometry and Integrable Systems"

The 5th International Workshop
Geometry of Submanifolds and Integrable Systems

November 26 - December 1, 2022

Abstracts

Plenary Speakers

November 26.

Shu-Cheng Chang (National Taiwan University, Taiwan)

Title: *The Legendrian Mean Curvature Flow*

Abstract

It is due to Colin-Giroux-Honda that, for fixing a Thurston–Bennequin invariant, a rotation number, and a knot type in contact 3-manifold, there are only finitely many Legendrian knot types realizing this data. However, it is proved by Ekholm-Etnyre-Sullivan that there is an infinite family of Legendrian embeddings of the n-sphere and n-torus into the standard contact Euclidean $(2n+1)$-space, $n>1$ that are not Legendrian isotopic even though they have the same rotation number and Thurston-Bennequin invariant. On the other hand, it is closely related between Legendrian and Lagrangian submanifolds. More precisely, the projection of the mean curvature of a Legendrian submanifold in the contactization of a Kaehler manifold coincides with the mean curvature of the projected Lagrangian submanifold in such a Kaehler manifold. In particular, the projection of an minimal Legendrian submanifold is an immersed minimal Lagrangian submanifold. In this lecture, we will address the recent result on the isotopic Legendrian problem and existence of minimal Legendrian submanifolds in η-Einstein Sasakian manifolds via the Legendrian mean curvature flow which is initiated by K. Smoczyk. Parts of the talk is based on the jointed work with Y. Han and C.-T. Wu.

Ryosuke Takahashi (Kyushu University, Japan)

Title: *A Nakai-Moishezon type criterion for supercritical deformed Hermitian-Yang-Mills equation*

Abstract

In this talk, we show that the solvability of the supercritical deformed Hermitian-Yang-Mills equation is equivalent to a certain algebraic positivity condition modeled on the Nakai-Moishezon ampleness criterion, which confirms the mirror version of the Thomas-Yau conjecture. This is a joint work with J. Chu (Peking University) and M.-C. Lee (Chinese University of Hong Kong).

Miyuki Koiso (Kyushu University, Japan)

Title: *A variational problem for developable surfaces with curved folds*

Abstract

Developable surfaces are surfaces which can be unfolded into the plane preserving the length of all curves on the surface. It is known that smooth developable surfaces in the three-dimensional Euclidean space are surfaces with zero Gaussian curvature, and they are generalized cylinders, generalized cones (away from apexes), and tangent developable surfaces. Since developable surfaces with curved folds are constructed by bending a flat sheet, they have many applications in manufacturing objects. In this talk, we discuss a variational problem for "pillow boxes" which are a specific class of closed developable surfaces with genus zero and with curved folds. For a given initial closed surface in a plane which consists of two rectangles with the same size whose boundaries are identified, we prove the existence and uniqueness of the maximizer of the enclosed volume (called the optimum solution) among piecewise smooth "cylindrical surfaces" which are isometric to the initial surface. Moreover, we give the explicit representation formula for the optimum solution, and show that there exists an isometric deformation from the initial planar surface to the optimum solution. (Joint work with Kento Okuda.)

November 27.

Pak Tung Ho (Sogang Univeristy, Korea)

Title: *The weighted Yamabe problem*

Abstract

In this talk, I will explain what the weighted Yamabe problem is, and mention some related results that Jinwoo Shin (KIAS) and I obtained.

Hiroyasu Sato (Nippon Institute of Technology, Japan)

Title: *The horosphere version of the Osserman conjecture and related topics*

Abstract

The horosphere version of the Osserman conjecture is stated as follows. Let M be a simply connected, complete manifold with no focal points. If every horosphere has constant principal curvatures (with multiplicities) and these principal curvatures (with multiplicities) do not depend on a choice of horospheres in M, then M is a rank-one symmetric space of non-compact type or a flat Euclidean space. In this talk, we introduce the above conjecture and related results. This work is based on the joint work with M. Itoh.

November 29.

Indranil Biswas (Tata Institute of Fundamental Research, India)

Title: *Moduli spaces of framed logarithmic and parabolic connections on a Riemann surface*

Abstract

We construct the moduli spaces of framed logarithmic connections and also framed parabolic connections on a Riemann surface with marked points. It is proved that these moduli spaces have algebraic symplectic structure. We also compute the degree of transcendence of the global algebraic functions on the moduli spaces. This is a joint work with Michi-aki Inaba, Arata Komyo and Masa-Hiko Saito.

Charles Ouyang (UMass Amherst, USA)

Title: *Compactifications of Hitchin components*

Abstract

Hitchin components are natural generalizations of the classical Teichmüller space. In the setting of $\mathrm{SL}(3,\mathbb{R})$, the Hitchin component parameterizes the holonomies of convex real projective structures, which are related to hyperbolic affine spheres. By studying Blaschke metrics, which are Riemannian metrics associated to hyperbolic affine spheres, along with their limits, we obtain a compactification of the $\mathrm{SL}(3,\mathbb{R})$ Hitchin component. We show the boundary objects are hybrid structures, which are in part flat metric and in part laminar. These hybrid objects are natural generalizations of measured laminations, which are the boundary objects in Thurston's compactification of Teichmüller space. Time permitting, we discuss the other rank 2 settings of $\mathbb{P}\mathrm{SL}(2,\mathbb{R}) \times \mathbb{P}\mathrm{SL}(2,\mathbb{R})$, $\mathrm{Sp}(4,\mathbb{R})$ and G_2.

Yuuki Tadokoro (National Institute of Technology, Kisarazu College, Japan)

Title: *Nonlinear $O(3)$ sigma model in discrete complex analysis*

Abstract

Mercat introduced a discrete Riemann surface with a discrete complex structure on bipartite quad-graphs on surfaces, and gave an algorithm to compute its period matrix. Bobenko and Günther treated Mercat's discrete Riemann surfaces more precisely and defined a new discrete complex structure using the medial graphs of the above quad-graphs.

We consider discrete Riemann surfaces on planar quad-graphs and its O(3) sigma model. We define the weighted discrete Dirichlet energy and area obtained from this sigma model, and explain the inequality between them. This is a discrete version of the inequality of Belavin and Polyakov. For general quad-graphs with orthogonal diagonals, we also show that the discrete (anti-)holomorphic function satisfies the Euler-Lagrange equations derived from the discrete energy.

On some special rhombic lattices, the discrete area can be regarded as the mapping degree of the discrete power function. Furthermore, we show that the discrete energy, discrete area, an

Euler-Lagrange equations tend to their continuous counterparts as the lattice spacing approaches zero.

This talk is based on joint work with Masaru Kamata and Masayoshi Sekiguchi.

Wayne Rossman (Kobe University, Japan)

Title: *Discrete Omega surfaces and their transformations*

Abstract

This talk lies within the field of discrete differential geometry (DDG), and will focus on recent joint work with Fran Burstall, Joseph Cho, Udo Hertrich-Jeromin and Mason Pember on discretization of Omega surfaces.

Such surfaces, in both the smooth and discrete settings, are a generalization of isothermic surfaces when viewed as Legendre immersions in 3-dimensional spaceforms (i.e. Euclidean, spherical and hyperbolic 3-spaces), as they have isothermic sphere congruences although they might not be isothermic themselves. Because of this, they have an underlying integrable-systems-based mathematical structure, and a resulting transformation theory.

Using Moebius and Lie sphere geometries, we will describe the equivalence of a number of approaches to discrete Omega surfaces, and also a satisfying transformation theory for them.

November 30.

Robert Kusner (Online, UMass Amherst, USA)

Title: *Steklov Eigenspaces and Free Boundary Minimal Surfaces*

Abstract

The coordinate functions on a free boundary minimal surface (FBMS) in the unit ball are Steklov eigenfunctions with eigenvalue 1. For many embedded FBMS in B^3 we show its first Steklov eigenspace coincides with the span of its coordinate functions, affirming a conjecture of Fraser & Li in an even stronger form.

Aya Ishizeki, Takeyuki Nagasawa (Saitama University, Japan)

Title: *Möbius energy for knots and links, and wave maps*

Abstract

The Möbius energy for knots can be decomposed into three parts. The first two are functionals which are still invariant under Möbius transformations, and the third one is an absolute constant. A similar decomposition also holds for the Möbius energy for links. The second part has a more complicated expression than the first one. Recently, the authors found its new and simple expression by using the Gauss map. Regarding it as a functional for maps from the 2-dimensional torus with Lorenz metric into a sphere, we observe that it is the energy of harmonic maps. That is, critical maps are wave maps. Here we consider knots and links which Gauss maps are wave maps.

Mao-Pei Tsui (National Taiwan University, Taiwan)

Title: *A new monotone quantity in mean curvature flow implying sharp homotopic criteria*

Abstract

We discuss a new monotone quantity in graphical mean curvature flows of higher codimensions. The submanifold deformed by the mean curvature flow is the graph of a map between Riemannian manifolds, and the quantity is monotone increasing under the area decreasing condition and suitable ambient curvature condition. The flow provides a natural homotopy of the corresponding map and leads to sharp criteria regarding the homotopic class of maps between complex projective spaces, and maps from spheres to complex projective spaces, among others. This is joint work with Chung-Jun Tsai and Mu-Tao Wang.

Takashi Sakai (Tokyo Metropolitan University, Japan)

Title: *Variational problems for integral invariants of the second fundamental form of a map between Riemannian manifolds*

Abstract

Motivated by integral geometry, we define integral invariants of a map between Riemannian manifolds by using invariant functions of the second fundamental form. Then we discuss variational problems for these integral invariants. We derive the first variational formulae for integral invariants defined from invariant homogeneous polynomials of degree two. Among these integral invariants, we show that the Euler–Lagrange equation of the Chern–Federer energy functional is reduced to a second order PDE. Then we give some examples of Chern–Federer submanifolds in Riemannian space forms. This talk is based on a joint work with Rika Akiyama and Yuichiro Sato.

Reiko Miyaoka (Tohoku University, Japan)

Title: *Various aspects of isoparametric hypersurfaces*

Abstract

In this talk, I will give various aspects of isoparametric hypersurfaces. Starting from the origin in geometric optics, I introduce isoparametric hypersurfaces, and state the classification. Then we consider the Gauss images of the hypersurfaces in the complex hyperquadric, and give a partial answer to the Arnold-Givental conjecture. As another aspect, the isoparametric functions are related to analytic problems such as the Pompeiu problem and the Schiffer problem. I mention recent informations on these problems.

December 1.

Emma Carberry (The University of Sydney, Australia)

Title: *Constant Mean Curvature Tori in* $\mathbb{R}^3$ *of Spectral Genus Two*

Abstract

A constant mean curvature (cmc) torus in $\mathbb{R}^3$ can be described in terms of a hyperelliptic spectral curve, whose genus g is at least two. The countably infinite space of examples studied by Wente and Abresch are contained in this simplest case of $g = 2$, but there are many more cmc tori in this simplest class. The object of this talk is to give a full account of the space of all cmc tori that have a spectral curve of genus $g = 2$. I shall describe how Whitham deformations can be used to give a complete account of the space of spectral data of real solutions of the sinh–Gordon equation of spectral genus 2. More precisely, I shall give a parameterisation of the closure of spectral data of constant mean curvature tori in $\mathbb{R}^3$ and analyse the boundary of this set. As a special case the Wente family, which is described by spectral data with real coefficients, is 1–dimensional, smooth and connected.

This is joint work with Martin Kilian, Sebastian Klein and Martin Schmidt.

Franz Pedit (UMass Amherst, USA)

Title: Higgs bundles, Monge-Ampere equations, affine spheres, and SYZ geometry

Abstract

Strominger-Yau-Zaslow suggest that Mirror Symmetry can be seen as T-duality on Calabi-Yau 3-folds fibered by special Lagrangian 3-tori which degenerate along a singular set modelled on a trivalent graph. We shall construct infinitely many such fibrations, singular over a Y-vertex, using the parabolic non-Abelian Hodge correspondence relating Higgs bundles, surface group representations, and equivariant harmonic maps. Along the way, we shall encounter rather classical geometric objects, the affine spheres introduced by the Blaschke school in the 1930s.

Short Communications (Parallel Session)

November 27.

Ryo Takenaka (Osaka Metropolitan University, Japan)

Title: *Fermionic character formulas of affine Lie algebras and related topics*

Abstract

Consider twisted affine Lie algebras obtained from simple Lie algebras and their automorphisms. The standard module is the integrable highest weight module. By using the vertex operator construction, we were able to obtain combinatorial bases of standard modules and some related algebras for twisted affine Lie algebras. In this talk, we introduce these results and possibility of application for several topics.

Shota Hamanaka (Mitsubishi Electric Corporation Advanced Technology R&D Center, Japan)

Title: C^0, C^1 *limit theorems for total scalar curvatures*

Abstract

We give some C^0 or C^1 limit theorems for total scalar curvatures. More precisely, we show that the lower bound of the total scalar curvatures on a closed manifold is preserved under the C^0 or C^1 convergence of the Riemannian metrics under some assumptions. Moreover, we give some counterexamples to the above theorems on an open manifold.

Yuki Koto (Kyoto University, Japan)

Title: *Analytic decomposition of quantum cohomology*

Abstract

Quantum cohomology of a smooth projective variety X is a formal deformation of the ordinary cohomology of X, and plays a very significant role in mirror symmetry.

Since QH is not functorial, it is interesting to investigate a relationship of quantum cohomology of different varieties. In this talk, we discuss quantum cohomology of a fiber bundle E over B whose fiber is a toric variety. We observe that, under a suitable assumption, quantum cohomology (D-module) of E is analytically decomposed into copies of that of B.

Homare Tadano (Yamaguchi University, Japan)
Title: *A Zoo of Bonnet-Myers Type Theorems*

Abstract

One of important issues in differential geometry is to investigate the relation between curvature and topology of Riemannian manifolds. I will introduce various Bonnet-Myers type theorems for complete Riemannian manifolds via m-Bakry-Émery Ricci curvature.

Denis Polly (Kobe University, Japan)
Title: *Representations of discrete Bryant type surfaces*

Abstract

The famed Weierstrass-Enneper representation for minimal surfaces in Euclidean space has an analogue in hyperbolic space: originally introduced for cmc 1 surfaces by Bryant in 1987 it can be used to generate a class of linear Weingarten surfaces called Bryant-type surfaces. This class covers the aforementioned cmc 1 surfaces but also flat fronts as well as constant harmonic mean curvature 1 surfaces.

In the realm of integrable discrete differential geometry, Bryant-type surfaces can be defined either via a discrete version of their representation or via discrete analogues of the linear Weingarten condition their smooth counterparts satisfy. In special instances it has been shown that these definitions coincide.

In this talk we provide an interpretation of the discrete Weierstrass-type representation for Bryant-type surfaces in terms of the transformation theory of discrete Omega surfaces. Further, we will prove that all Bryant-type surfaces, as defined via their linear Weingarten condition, can be generated via this representation.

Kuang-Ru Wu (Academia Sinica, Taiwan)
Title: *Positively curved Finsler metrics on vector bundles*

Abstract

While the equivalence between ampleness and positivity holds for vector bundles of rank one, its higher rank counterpart known as Griffiths' conjecture is still open. There is also a similar but weaker conjecture by Kobayashi who proposed to use Finsler rather than Hermitian metrics to study the equivalence. We will review these two conjectures and state our progress. One of our results is that Kobayashi positivity implies ampleness and convex Kobayashi positivity. We will also discuss how to prove Kobayashi positivity for ample vector bundles with additional curvature assumptions.

Joseph Cho (TU Wien, Austria)

Title: *Closed discrete Darboux transforms*

Abstract

In this talk, we discuss the conditions to obtain closed Darboux transforms of discrete isothermic surfaces. We test the robustness of our theory by creating concrete examples. This talk is based on the joint work with Katrin Leschke (University of Leicester) and Yuta Ogata (Kyoto Sangyo University).

Brian Harvie (National Taiwan University, Taiwan)

Title: *The Bartnik quasi-local mass of small geodesics balls*

Abstract

A fundamental problem in Einstein's theory of general relativity is to find a suitable notion of the mass of a space-like, time-symmetric hypersurface in a spacetime. The Arnowitt-Deser-Misner (ADM) mass manages to measure the total mass of such a slice in an isolated gravitating system, but determining the mass contained within a compact region via a geometric quantity is more difficult. R. Bartnik proposed a definition for the mass of a compact domain that utilizes the ADM mass and has an important relationship with equilibrium solutions to the Einstein equations called "static" spacetimes.

In this talk, I will discuss recent work with Prof. Ye-Kai Wang about estimating the Bartnik mass of small geodesic balls in Riemannian 3-manifolds by studying their static extensions, which builds upon earlier work by David Wiygul. I would like to make the talk accessible to differential geometers without a relativity background, and so I will focus mostly on exposition.

Artur Sergyeyev (Online, Silesian University in Opava, Czech Republic)

Title: *From contact geometry to multidimensional integrable systems*

Abstract

In this talk we showcase a novel application of three-dimensional contact geometry, where it helps answering a longstanding question of just how exceptional are partial differential systems in four independent variables that are integrable in the sense of soliton theory. It turns out that such systems are far more numerous than it was believed, and we provide an effective explicit construction, involving contact vector fields, for a large class of systems in question along with their Lax pairs. As a byproduct, we present a first example of an integrable partial differential system in four independent variables with a nonisospectral Lax pair which is algebraic, rather than rational, in the spectral parameter.

Tetsuya Nakamura (UMass Amherst)

Title: *$Sp(C^4)$ Transformations of CMC 1 Surfaces in H^3*

Abstract

Constant mean curvature (CMC) 1 surfaces in 3 dimensional hyperbolic space H^3 are represented by a null curve immersion into a 3 dimensional quadric in CP^4 . This representation is due to Bryant. Linear group $SO(C^5)$, which leaves the quadric invariant, naturally acts on null curve immersions. In this talk, we will see with some examples that this action gives rise to (locally defined) non-trivial deformations of the corresponding CMC 1 surface in H^3. Furthermore the standard action of its spin group $Sp(C^4)$ on C^4 is understood as a linear transformation of spinors associated with the CMC 1 surface. From this perspective, we will observe that the transformation is globally defined if we consider a special class of CMC 1 surfaces, namely of genus 0 and with smooth ends. In addition to it, the transformation preserves the Willmore energy of the surface.

November 28.

Ye-Kai Wang (Online, National Yang Ming Chiao Tung University, Taiwan)

Title: *A rigidity result for surfaces in Schwarzschild spacetime*

Abstract

We study spacelike surfaces in Schwarzschild spacetime whose mean curvature vector has constant norm. When the surface lies in the standard null hypersurface, the problem is closely related to the Liouville and Obata Theorem in conformal geometry. We show that such surfaces must lie in a time slice. The talk is based on the joint work with Po-Ning Chen at UC Riverside.

Albert Wood (National Taiwan University, Taiwan)

Title: *Cohomogeneity-One Lagrangian Mean Curvature Flow*

Abstract

Lagrangian mean curvature flow is the name given to the observation that Lagrangian submanifolds of Calabi-Yau manifolds are preserved under mean curvature flow. This observation gave rise to a conjecture of Thomas-Yau, which states that assuming a ‘stability’ condition on the Lagrangian, the flow should converge to a unique volume-minimising representative: a special Lagrangian. Since mean curvature flow typically forms finite-time singularities, a surgery procedure must be defined to resolve the conjecture, and an understanding of the possible singularity models is a vital first step.

In this work, we study Lagrangians that are invariant with respect to a group action respecting the Calabi-Yau structure with $(n-1)$-dimensional orbits. Such Lagrangians must be contained in an $(n+1)$-dimensional submanifold, a level set of the moment map, and taking the symplectic quotient produces a curve in a 2-manifold. Lagrangian mean curvature flow may therefore be studied via a related curve shortening flow, which we show does not depend on the group action. By this method, we are able to classify cohomogeneity-one shrinking and expanding solitons, as well as fully classify singularities in the case of the zero level set.

Taiki Yamada (Shimane University, Japan)

Title: *Asymmetric allocation rule depending on graph structure*

Abstract

This talk will be given in collaboration with Taisuke Matsubae (Chuo University), and will be based on arXiv:2110.06506. The primary focus in cooperative game theory concerns the sharing or distribution of the worth gained from a cooperation. To this end, we usually consider Shapley value that introduced in 1953. However, since this value does not reflect the structure of the graph, we define a new allocation rule by using the Shapley value. In this talk, we will compare the Shapley values with our allocation rule and also present its relevance to discrete differential geometry.

Yen-Chang Huang (National University of Tainan, Taiwan)

Title: *Some integral geometric formulas in the Heisenberg groups*

Abstract

In this talk, I will introduce our recent works related to some integral formulas in Cauchy Riemann manifolds (CR manifolds). The classical Cauchy surface area formula sates that the surface area of any compact hypersurface in the n-dimensional Euclidean space equals the average of all projected surface areas onto the $(n-1)$-subspaces; the similar result can be generalized to the three-dimensional Heisenberg group, which is regard as a one-dimensional CR manifold with zero Tanaka-Webster curvature. In contract to the Euclidean case, the properties of the Pansu spheres play the key ingredient in the proofs instead of the round spheres. The result supports that the Pansu spheres may be a more appropriate model in studying the geometry of the Heisenberg groups rather than the round spheres.

Chih-Wei Chen (National Sun Yat-sen University, Taiwan)

Title: *Hessian estimators on data manifolds*

Abstract

Given a function defined on a data set, there have been several ways to estimate the Hessian of it. Although the estimators can be quite accurate, their constructions are all based on certain grid structures. Without using any grid assumption, we construct a Hessian estimator which converges to the continuous Hessian tensor when the data points are densely sampled from a data manifold. This is a joint work with Dr. Hau-Tieng Wu.

Abhitosh Upadhyay (Indian Institute of Technology Goa, India)

Title: *On compact embedded Weingarten hypersurfaces in warped products*

Abstract

The well-known Alexandrov theorem ensures that a closed embedded hypersurface of the Euclidean space with constant mean curvature must be a round sphere. It was proved by Montiel and Ros that the Alexandrov theorem for mean curvature as well as higher order mean curvatures is also true for hypersurfaces of hyperbolic spaces and half-spheres. Recently, de Lima proved a comparable result for the so called linear Weingarten hypersurfaces satisfying $H_r = aH + b$ for two real constants $a \geq 0$ and $b > 0$, where H and H_r are respectively the mean curvature and the r-th mean curvature of the hypersurfaces. In this talk, I will discuss that Lima's result also holds for a large class of warped products which contains in particular the hyperbolic spaces and the half-spheres.